

Problem Set: Accuracy, Precision and Error Analysis

Learning Objectives

- Determine the absolute and percentage error (or % relative error) in addition, subtraction, multiplication and division calculations.
- Use a percentage error (or % relative error) to determine the number of significant figures that should be reported in a measurement

You should review SRM01 posted on Bruin Learn or the following web site on the rules of significant figures before working on the problem set.

http://chemcollective.org/activities/tutorials/stoich/significant_figures

Practice problems set is NOT a homework assignment. The problems set is designed to give students additional practice on learning the concepts and problems solving. You should work on the questions first before checking the answer key. This is the only way to make sure you understand the concepts we discussed in lecture. Simply reading the answer key is not a good way to practice concepts!

1. Complete the following table (the first entry is provided as an example). Which entry has the highest % relative error? Briefly explain your reasoning.

Measured Value	Scientific Notation	Absolute Error of “±1” (LAST significant digit)	% Relative Error (Absolute Error ÷ Measurement)
10. mL	1.0×10^1	0.1×10^1	$0.1 \times 10^1 / 1.0 \times 10^1 = 10\%$
0.0020 g			
10.0 mL			
0.0200 seconds			
8.4 L			
10.00 mL			
0.2000 M			
23.44 mL			

2. Determine the % relative error **and** absolute error that arise from the following calculations. Final value should have the correct number of significant figures. If no error is given for a measurement, you may assume an absolute uncertainty of “1” in the last significant digit of the measurement.

Example: A value of 0.15 has a minimum absolute error of 0.01 (i.e. uncertainty of “1” in the last significant digit)

Refer to the appropriate % relative error equation **and** absolute error equation provided on page 5 (or lecture #1) depending on the mathematical operation when solving for this question.

- a. $19.32 \text{ cm} \times 98.71 \text{ cm}$
- b. $(0.4245 \pm 0.0002 \text{ M}) \times (15.65 \pm 0.02 \text{ mL}) \div (0.5052 \pm 0.0004 \text{ M})$
- c. $(30.078 \pm 0.003) - (20.174 \pm 0.001) + (9.813 \pm 0.002)$
- d. $(13.65 \pm 0.02 \text{ mL}) + (6.03 \pm 0.02 \text{ mL})$

3. A student recorded a measured value as 2.056g. Determine the minimum % relative error in the measured value.
4. A student recorded a value of 2.6547282 based on the display on a calculator. The value has a relative error of 4%. How should the student report the value based on the % error in the measurement.
5. Calculate the percent relative average deviation (%RAD) of the following set of numbers. Report the average with the correct significant figures based on the value for the relative average deviation.

0.773, 0.729, 0.779, 0.762, 0.770

6. Apply the critical ratio test to the 5 data points in Question 4. Can the data point “0.729” be discarded on statistical grounds?

Note: Refer to pages IV.6 – 7 in the manual for explanation and example on critical ratio test. Critical ratio test is one method to reject a scientific measurement based on the critical ratio in a set of measurement.

Answer Key

1.

Measured Value	Scientific Notation	Absolute Error if ± 1 in last significant digit	% Relative Error
10. mL	1.0×10^1	1 or 0.1×10^1	$(0.1 \times 10^1 / 1.0 \times 10^1) \times 100\% = 10\%$
0.0020 g	2.0×10^{-3}	0.0001	5%
10.0 mL	1.00×10^1	0.1	1%
0.0200 seconds	2.00×10^{-2}	0.0001	0.5%
8.4 L	8.4×10^0	0.1	1%
10.00 mL	1.000×10^1	0.01	0.1%
0.2000 M	2.000×10^{-1}	0.0001	0.05%
23.44 mL	2.344×10^1	0.01	0.04%

The first entry has the highest % relative error because it is the least precisely known value in the table.

2. Use the equations we introduced in lecture 1 on error analysis to calculate the % relative error and absolute error. The equations that you use should be based on the mathematical operation.

(a) Relative percent error: 0.06% (see equation below)

$$100\% \times \left(\frac{\pm 0.01 \text{ cm}}{19.32 \text{ cm}} + \frac{\pm 0.01 \text{ cm}}{98.71 \text{ cm}} \right)$$

Here we use the relative error equation for "Multiplication/Division"

Note: Absolute error = Final quantity $\times$ Relative error = $(19.32 \text{ cm} \times 98.71 \text{ cm}) \times 0.0006$

Absolute error: $\pm 1 \text{ cm}^2$ (or you can report the error as $1907 \text{ cm}^2 \pm 1 \text{ cm}^2$)

(b) Relative percent error: 0.3% (see equation below)

$$100\% \times \left(\frac{\pm 0.0002 \text{ M}}{0.4245 \text{ M}} + \frac{\pm 0.02 \text{ mL}}{15.65 \text{ mL}} + \frac{\pm 0.0004 \text{ M}}{0.5052 \text{ M}} \right)$$

Here we use the relative error equation for "Multiplication/Division"

Note: Absolute error = Final quantity $\times$ Relative error = $(0.4245 \text{ M} \times 15.65 \text{ mL} \div 0.5052) \times 0.003$

Absolute error: $\pm 0.03 \text{ mL}$ or $\pm 0.04 \text{ mL}$

(or you can report the error as $13.15 \pm 0.03 \text{ mL}$ or $13.15 \pm 0.04 \text{ mL}$)

(c) Relative percent error: 0.03% (see equation below)

$$100\% \times \left(\frac{\pm 0.003}{19.717} + \frac{\pm 0.001}{19.717} + \frac{\pm 0.002}{19.717} \right)$$

Here we use the relative error equation for "Addition / Subtraction"

Notice that we always sum the relative errors even if subtraction is involved in the equation.

Absolute error: ± 0.006 (or 19.717 ± 0.006)

2. Use the equations we introduced in lecture 1 on error analysis to calculate the % relative error and absolute error. The equations that you use should be based on the mathematical operation.

(d) Relative percent error: 0.2% (see equation below)

$$100\% \times \left(\frac{\pm 0.02 \text{ mL}}{19.68 \text{ mL}} + \frac{\pm 0.02 \text{ mL}}{19.68 \text{ mL}} \right)$$

Here we use the relative error equation for “Addition” (Same as the addition/subtraction category!)

Absolute error: $\pm 0.04 \text{ mL}$ (or $19.68 \pm 0.04 \text{ mL}$)

3. **Minimum % relative error = (Minimum absolute error/Measurement) $\times$ 100%**
 $= (\pm 0.001 \text{ g} / 2.056 \text{ g}) \times 100\% = 0.05\% \text{ or } \pm 0.05\%$

An absolute error of “1” in the last significant digit constitutes the minimum absolute scientific error!

4. $(\text{Absolute Error} / \text{Measurement}) \times 100\% = \% \text{ Relative Error}$
 $(\text{Absolute Error} / 2.6547282) \times 100\% = 4\%$
 Absolute Error = 0.1

Therefore, the student can only the value to one decimal place (i.e. 2.7 ± 0.1)

5. **Calculated average = 0.763**

% RAD = 1.8%

You should write out all the terms and you will see why the %RAD only contains 2 significant figures! If necessary, review the rule of significant figures using the online tutorial posted on the beginning of this problem set. Use the following equation when calculating %RAD. In the equation, “ $\bar{x}$ ” refers to the average value of the data set. “ x_i ” refers to each individual data point. “ n ” is the total number of data points. “ n ” is an integer and should NOT be considered as part of the significant figures. “ Δx ” is the relative deviation in the data. RAD is defined as relative deviation divided by average value.

$$\overline{\Delta x} = \frac{\sum_{i=1}^n |(x_i - \bar{x})|}{n}$$

$$\% \text{RAD} = 100\% \times \frac{\overline{\Delta x}}{\bar{x}}$$

- 6.

$x_q = 0.729$; $x_n = 0.762$; $x_f = 0.779$

$$R_f = \left| \frac{x_q - x_n}{x_q - x_f} \right| = \left| \frac{0.729 - 0.762}{0.729 - 0.779} \right| = 0.66$$

Since the value of R_f is greater than the critical ratio for a sample with five data points (0.642), the value 0.729 can be discarded based on statistical grounds. Refer to pages IV.6 – 7 in the manual for explanation and example on critical ratio test.

Estimating Uncertainties by Using Mathematical Models

When calculating the % relative error (or sometimes refer to as % error), you need to know the following mathematical equations and know how to apply them.

E in here is the experimental quantity being calculated based on the measured values of A and B.

A is the quantity being measured in the laboratory. B is another quantity being measured in the laboratory

Suppose each quantity A & B contains an uncertainty of ΔA & ΔB respectively. ΔA & ΔB are the absolute errors in A & B respectively (i.e. $A \pm \Delta A$ and $B \pm \Delta B$)

To estimate the uncertainty in the *experimental* quantity E, you need to know the following 4 equations depending on the type of mathematical operation.

One can extend the following equations to more than 2 terms in the calculation.

(I) Addition AND/OR Subtraction (equation only involves “+” and/or “-”)

$E = A+B$ OR $E=A-B$ (depending on the actual mathematical equation)

(a) Relative error in E is:

$$\frac{\Delta E}{E} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

(b) Absolute error in E is:

$$\Delta E = \Delta A + \Delta B$$

(II) Multiplication AND/OR Division (equation only involves “×” and/or “÷”)

$E = A \times B$ OR $E=A \div B$ (depending on the actual mathematical equation)

(a) Relative error in E is:

$$\frac{\Delta E}{E} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

(b) Absolute error in E is:

$$\Delta E = \left(\frac{\Delta A}{A} + \frac{\Delta B}{B} \right) \times E$$

EXAMPLE #1:

Subtraction between 2 values

• • • • ± • • • • - • • • • ± • • • •

% Relative Error

$$= \left(\left(\frac{\pm 0.02}{4.65} \right) + \left(\frac{\pm 0.02}{4.65} \right) \right) \times 100\%$$

Note: The term “ $\pm 0.02/4.65$ ” refers to as relative error. We always sum the relative error even when dealing with subtraction between 2 values!

$$= 0.9\% \text{ or } \pm 0.9\%$$

The value of 4.65 came from $5.05 - 0.40$.

“ ± 0.02 ” is the absolute error for each of the numerical quantity (i.e. 5.05 and 0.40).

“ $\pm$ ” sign is just a symbolic notation for error. It does not affect the calculation.

Absolute Error

Absolute error = Relative error (in non % value) $\times$ Final Calculated Quantity

$$= \pm 0.009 \times 4.65 \text{ OR } 0.009 \times (5.05 - 0.40)$$

$$= \mathbf{0.04 \text{ or } \pm 0.04}$$

You can also report the absolute error as 4.65 ± 0.04 . Notice that both the final quantity and the absolute error has the same number of decimal places (NOT the same number of significant figures). This means the final value has a range between 4.61 and 4.69.

EXAMPLE #2:

Multiplication between 2 values

$$\dots \pm \dots \times \dots \pm \dots$$

% Relative Error

$$= \left(\left(\frac{\pm 0.02}{5.00} \right) + \left(\frac{\pm 0.0002}{0.4000} \right) \right) \times 100\%$$

$$= 0.5\% \text{ or } \pm 0.5\%$$

Absolute Error

Absolute error = Relative error (in non % value) $\times$ Final Calculated Quantity

$$= \pm 0.005 \times 2.00 \text{ OR } 0.005 \times (5.00 \times 0.4000)$$

$$= \mathbf{0.01 \text{ or } \pm 0.01}$$

You can also report the absolute error as 2.00 ± 0.01 . Notice that both the final quantity and the absolute error has the same number of decimal places (NOT the same number of significant figures). This means the final value has a range between 1.99 and 2.01.